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COMMENT

A non-Abelian Aharonov–Bohm effect in the framework of pseudoclassical mechanics

Luiz C L Botelho† and J C de Mello‡

† Centro Brasileiro de Pesquisas Fisicas, CBPF/CNPq Rua Xavier Sigaud, 150, 22290, Rio de Janeiro, RJ, Brasil

‡ Departamento de Física dos Sólidos, Instituto de Física Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brasil

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Abstract. We analyse a non-Abelian Aharonov-Bohm effect in the framework of pseudoclassical mechanics.

For a long time there has been much discussion in the literature about the so-called Aharonov–Bohm effect (Aharonov and Bohm 1959). In the Wu–Yang paper (1975) a non-Abelian generalisation of this effect was proposed. We aim in this comment to analyse a similar phenomenon in the framework of pseudoclassical mechanics (Barducci *et al* 1977, Berezin and Marinov 1977), showing that this effect appears even for a scalar charged particle (see, e.g. Horváthy and Kollár 1984, Horváthy 1984 for a similar conclusion) and that this is a manifestation of the internal degree of freedom associated to the colour degree.

We consider a 'Gedanken' experiment as follows. A colour $O(N)$ charged, spinless, particle is allowed to move from a spacelike point A to a spacelike point B, along a definite path C_{AB} connecting these points in a region R where there is a non-zero non-Abelian $O(N)$ gauge field with the property that its associated field strength vanishes. We remark that the region R can have an arbitrary topology.

In the framework of pseudoclassical mechanics, the motion of a colour (spinless) charged particle is characterised by the usual (relativistic) vector position $\chi^\mu(\xi) = (\boldsymbol{\chi}(\xi), \chi_0(\xi))$ added with a set of Grassmann complex variables $\{\theta_l(\xi), \theta_l^*(\xi)\}$, where $\xi_i \leq \xi \leq \xi_f$ denotes a parameter describing the evolution of the system and $l = 1, \ldots, N$ is the number of generators of the $O(N)$ Lie algebra (Barducci *et al* 1977). As the $\chi^\mu(\xi)$ trajectory is given by the path C_{AB}, we consider solely that the evolution of the colour degrees $\theta^{(AB)}(\xi)$ and its associated equation of motion is given by (Barducci *et al* 1977)

$$\begin{aligned} &\mathrm{d}\theta^{(AB)}(\xi)/\mathrm{d}\xi = -\mathrm{i}gA_\mu(\chi^\mu(\xi))\theta^{(AB)}(\xi) \\ &\theta^{(AB)}(\xi_0) = \chi_0. \end{aligned} \tag{1}$$

Note that the ordinary differential equation above possesses $O(N)$ matrix indices. Then the solution is given by the well known Wu–Yang factor defined by the gauge field $A_\mu(\chi)$ $(\chi \in R)$ and the path C_{AB}:

$$\{\chi_\mu(\xi): \xi_i \leqslant \xi \leqslant \xi_f\}:$$
$$\theta_j^{(AB)}(\xi) = \left\{ \boldsymbol{P}\left[\exp\left(ig \int_0^{\xi} A_\mu(\chi^\mu(\sigma)\mathring{\chi}^\mu(\sigma))\right)\right]\right\}_{jk} (\chi_0)_k \tag{2}$$

where (j, k) are $O(N)$ indices. As a result of solution (2) we have

$$\theta_j^{(AB)}(\xi_f) = \left\{ \boldsymbol{P}\left[\exp\left(ig \int_{C_{AB}} A\, d\chi^\mu\right)\right]\right\}_{jk} (\chi_0)_k \tag{3}$$

where we introduce the abbreviated notation $\int_{C_{AB}} A_\mu\, d\chi^\mu$ to denote the path integration in (2).

Now, after the particle has described the path C_{AB}, we allow the particle to return to the initial point A by a spacelike path

$$C_{BA}: \{\bar{\chi}_\mu(\bar{\xi}): \xi_f \leqslant \bar{\xi} \leqslant \xi_i\}.$$

Proceeding as above, we consider the $\bar{\xi}$ $O(N)$ colour degree associated evolution:

$$d\tilde{\theta}^{(BA)}(\bar{\xi})/d\bar{\xi} = -igA_\mu(\bar{\chi}_\mu(\bar{\xi}))\tilde{\theta}^{(BA)}(\bar{\xi})$$
$$\tilde{\theta}^{(BA)}(\xi_0) = \theta^{(AB)}(\xi_f). \tag{4}$$

The solution of (4) is given by

$$\tilde{\theta}_j^{(BA)}(\xi_i) = \left\{ \boldsymbol{P}\left[\exp\left(ig \int_{C_{BA}} A_\mu\, d\chi^\mu\right)\right]\right\}_{jk} (\theta^{(AB)}(\xi_f))_k. \tag{5}$$

Now if we compare the values of the $O(N)$ colour degrees at the point A after the particle has described the closed path $C_{AB} \cup C_{BA} = C_{AA}$, we see that they do not coincide since they are related by the Wu-Yang factor defined by the non-zero gauge field A_μ and the closed path C_{AA}, i.e.

$$\tilde{\theta}^{(BA)}(\xi_i) = \boldsymbol{P}\left[\exp\left(ig \int_{C_{AA}} A_\mu\, d\chi^\mu\right)\right]\theta^{(AB)}(\xi_i). \tag{6}$$

We have shown that the outcome of the proposed 'Gedanken' experiment, analysed on the framework of pseudoclassical mechanics, depends fundamentally on the Wu-Yang non-integrable phase factor in equation (6) as predicted earlier by Wu and Yang (1975).

Some related studies exploiting the relation between gauge fields and particle internal structure have been made by Horváthy (1984) and Horváthy and Kollár (1984).

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